

Perton First School



Concrete Pictoral Abstract

Progression



This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added. It is a working document and will be revised and amended as necessary.

RERTON FIRST SCHOOL





Concrete Pictorial Abstract

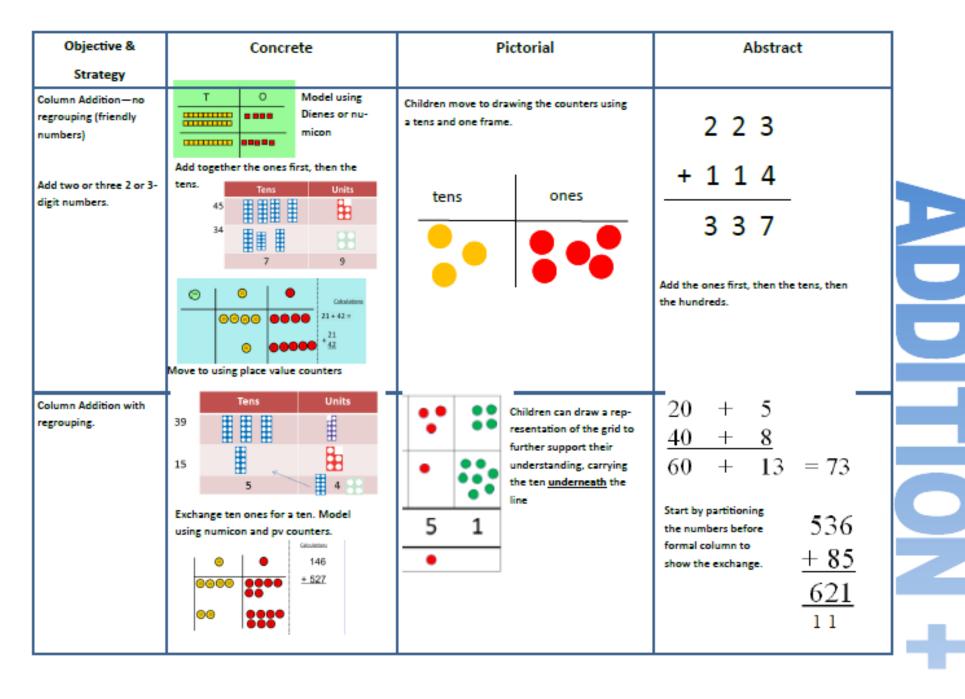
Progression in Addition

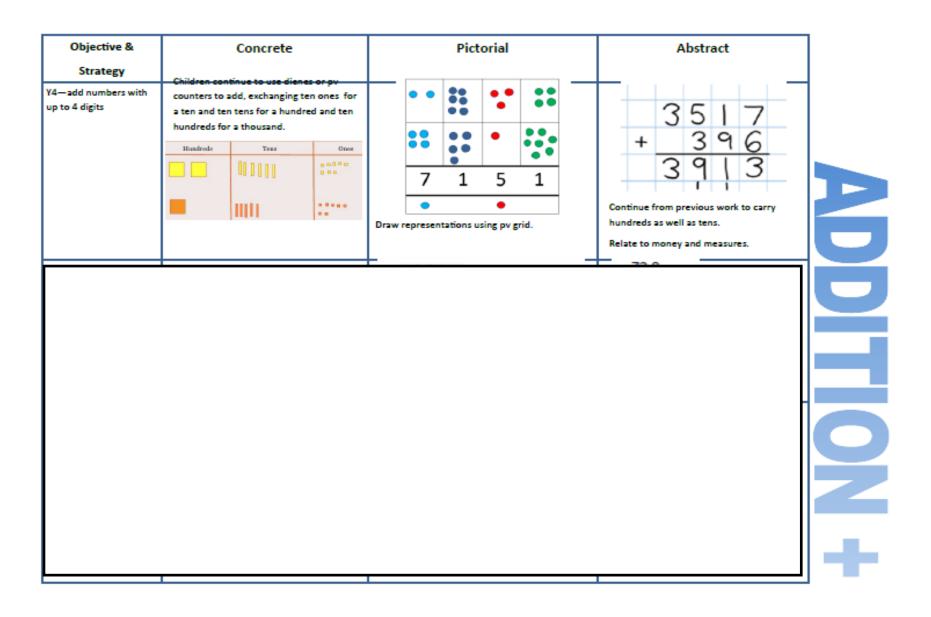


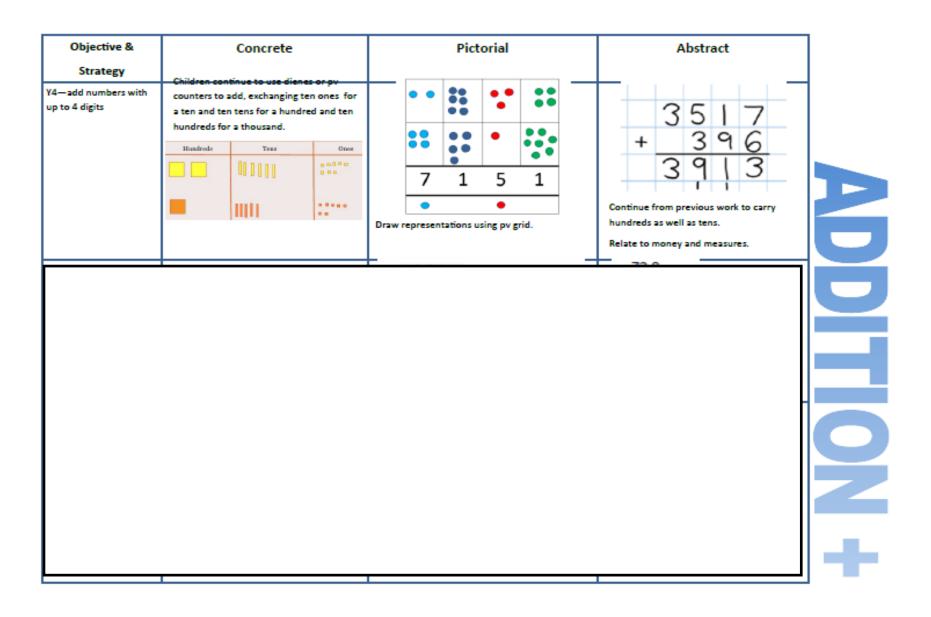
Objective & Strategy	Concrete	Pictorial	Abstract	
Combining two parts to make a whole: part- whole model	Use part part whole model. Use cubes to add two numbers together as a group or in a bar.	Use pictures to add two numbers together as a group or in a bar.	4 + 3 = 7 Use the part-part whole diagram as shown above to move into the abstract.	3
Starting at the big- ger number and counting on	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17 10 11 12 13 14 15 16 17 16 19 20 Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.	
Regrouping to make 10. This is an essential skill for column addition later.	Start with the bigger number and use the smaller number to make 10. Use ten frames.	Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10. 9 + 5 = 14	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?	
Represent & use number bonds and related subtraction facts within 20	2 more than 5.	Step 2 mare ton	Emphasis should be on the language '1 more than 5 is equal to 6.' '2 more than 5 is 7.' '8 is 3 more than 5.'	

Objective &	Concrete	Pictorial	Abstract
Strategy Adding multiples of	50= 30 = 20		20 + 30 = 50
ten	11111	2 lone + F lonetone	70 = 50 + 20 40 + = = 60
	Model using dienes and bead strings	30 + 60 = Use representations for base ten.	40 7 11 = 00
Use known number facts Part part whole	Children explore ways of making numbers within 20	20 C 	1 + 1 = 16
		+= 20	
Using known facts	+ =	∵ + ÷ = .\$	3 + 4 = 7
	חחח חחח חחחחחח	+ =	leads to
		+ = = =	30 + 40 = 70
			leads to
		Children draw representations of H,T and O	300 + 400 = 700
Bar model		茶茶茶茶茶茶 茶 茶 茶	23 25
		00000000000	?
	3+4=7	7 + 3 = 10	23 + 25 = 48









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Progression in Subtraction



Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, counters, cubes etc to show how objects can be taken away. 6-4 = 2		7—4 = 3
	4-2=2	15 - 3 = 12 Cross out drawn objects to show what has been taken away.	16—9 = 7
Counting back	Move objects away from the group, counting backwards. Move the beads along the bead string as you count backwards.	Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?
Find the Difference	Compare objects and amounts 7 'Seven is 3 more than four' 4 'I am 2 years older than my sister' 3 Fences 2 Lay objects to represent bar model.	Count on using a number line to find the difference.	Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister.?

2 BTRACTION

Objective & Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 Part Part Whole model	Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the arts, what s the other part? 10—6 = 4	Use pictorial representations to show the part.	Move to using numbers within the part whole model. 5
Make 10	Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	13—7 13—7 = 6 Jump back 3 first, then another 4. Use ten as the stopping point.	16—8 How many do we take off first to get to 10? How many left to take off?
Bar model	5-2=3		8 2 10 = 8 + 2 10 = 2 + 8 10 - 2 = 8 10 - 8 = 2

2 BTRACTION

Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'	20 – 4 =	20—4 = 16
Partitioning to sub- tract without re- grouping. 'Friendly numbers'	Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off. 43—21 = 22	43—21 = 22
Make ten strategy Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.	34—28 Use a bead bar or bead strings to model counting to next ten and the rest.	76 80 90 93 'counting on' to find 'difference' Use a number line to count on to next ten and then the rest.	93—76 = 17



Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	47—32 Use base 10 or Numicon to model	Darw representations to support under- standing	$47 - 24 = 23$ $-\frac{40 + 7}{20 + 3}$ Intermediate step may be needed to lead to clear subtraction under- standing. 32 -12 20
Column subtraction with regrouping	Tens Units	45 -29 Tens 10 nes 16	8 3 6 - 2 5 4 * 5 8 2 8 3 6 - 2 5 4 * 5 8 2 8 2 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8
	Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into tten ones. Use the phrase 'take and make' for exchange.	Children may draw base ten or PV counters and cross off.	7 2 8 - 5 8 2 = 1 4 6 Then move to formal method. 5 8 2 1 4 6
Subtracting tens and ones Year 4 subtract with up to 4 digits. Introduce decimal subtraction through context of money	234 - 179	Children to draw pv counters and show their exchange—see Y3	2 7 5 4 - 1 5 6 2 1 1 9 2 Use the phrase 'take and make' for exchange

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Concrete Pictorial Abstract

Progression in Multiplication



Objective &	Concrete	Pictorial	Abstract
Strategy			
Doubling	Use practical activities using manipultives including cubes and Numicon to demonstrate doubling	Double 4 is 8	Partition a number and then double each part before recombining it back together. 16 10 6 12 20 + 12 = 32
Counting in multi- ples	Count the groups as children are skip counting, children may use their fingers as they are skip counting.	Children make representations to show counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
Making equal groups and counting the total	Use manipulatives to create equal groups.	Draw to show 2 x 3 = 6 Draw and make representations	2 x 4 = 8

Objective &	Concrete	Pictorial	Abstract	
Strategy				
Repeated addition	Use different objects to add equal groups	Use pictorial including number lines to solve probl There are 3 sweets in one bag. How many sweets are in 5 bags altogether? 3+3+3+3 15	Write addition sentences to describe objects and pictures. 2 + 2 + 2 + 2 + 2 = 10	
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.	Draw representations of arrays to show understanding	3 x 2 = 6 2 x 5 = 10	PLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters. 40 + 12 = 52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together. 16 10 6 1x2 1 x2 20 + 12 = 32
Counting in multi- ples of 2, 3, , 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models. 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40	Number lines, counting sticks and bar models should be used to show representation of counting in multiples. 3 3 3 3 3	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30

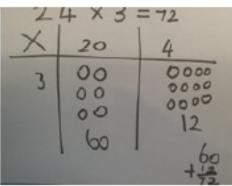
Objective & Strategy	Concrete	Pictorial	Abstract	
Multiplication is commutative	Create arrays using counters and cubes and Numicon. Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.	Use representations of arrays to show different calculations and explore commutativity.	12 = 3 × 4 12 = 4 × 3 Use an array to write multiplication sentences and reinforce repeated addition. 00000 00000 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15	
Using the Inverse This should be taught alongside division, so pupils learn how they work alongside each other.		8 x =	2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8 ÷ 2 Show all 8 related fact family sentences.	

Objective & Concrete Strategy Show the links with arrays to first intro-Grid method duce the grid method 4 rows of 10 4 rows of3 Move onto base ten to move towards a more compact method. 4 rows of 13 Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows Calculations 4 x 126 Fill each row with 126 Calculations 4×126 Add up each column, starting with the ones making any exchanges needed Then you have your answer.

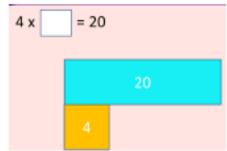
Pictorial

Children can represent their work with place value counters in a way that they understand.

They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.



Bar model are used to explore missing numbers



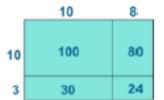
Abstract

Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

×	30	5
7	210	35

$$210 + 35 = 245$$

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.





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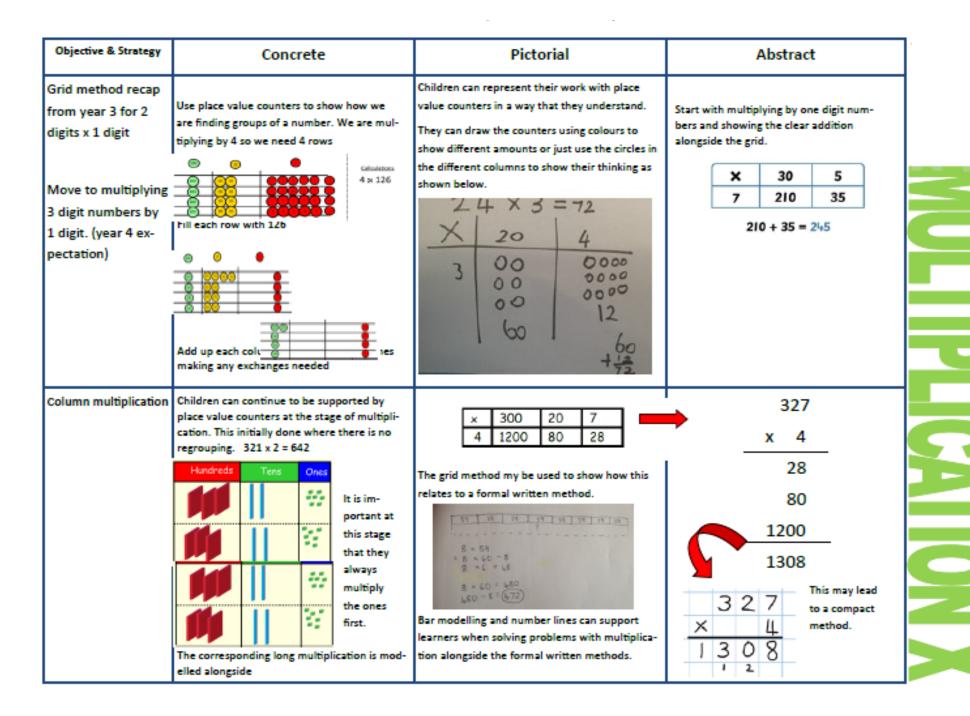


Concrete Pictorial Abstract

Progression in Division



Objective & Strategy	Concrete	Pictorial	Abstract	
Division as sharing		Children use pictures or shapes to share quanti- ties.	12 shared between 3 is	
Use Gordon ITPs for modelling		\$\frac{3}{3}\frac{3}\frac{3}{3}\frac{3}{3}\frac{3}{3}\frac{3}{3}\frac{3}{3}\frac{3}{3}\frac{3}{3}\frac{3}{3}\frac{3}{3}\frac{3}{3}\frac{3}{3}\frac{3}{3}\frac{3}{3}\frac{3}{3}\frac{3}{3}\frac{3}{3}\frac{3}{3}\frac{3}\frac{3}{3}\frac{3}{3}\frac{3}{3}\frac{3}{3}\frac{3}{3}\frac	4	_
		Sharing:		
	10	12 shared between 3 is 4		•
	I have 10 cubes, can you share them equally in 2 groups?			



Objective &	Concrete	Pictorial	Abstract	
Division as sharing	I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities. 8 + 2 = 4 Children use bar modelling to show and support understanding.	12 ÷ 3 = 4	
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.	Use number lines for grouping 12 ÷ 3 = 4 Think of the part as a whole, applications the number of groups you are dividing by and work out how many would be within each group. 20 ÷ 5 = ? 5 x ? = 20	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?	

Objective &	Concrete	Pictorial	Abstract
Strategy			
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding. 24 divided into groups of 6 = 4 96 ÷ 3 = 32	Continue to use bar modelling to aid solving division problems. 20 ? 20 ÷ 5 = ? 5 x ? = 20	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

Objective &	Concrete	Pictorial	Abstract
Strategy			
Strategy Division with remainders.	Divide objects between groups and see how much is left over Example without 40 + 5 Ask "How many Example with re 38 + 6	5s in 40?" 5+5+5+5+5+5+5 = 8 f	
	jumps can be rec	ers, when it becomes inefficient to count in single moorded using known facts.	uiupies, viiggei

Objective &	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division	3 2 3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups. Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. 2 1 8 3 4 8 7 2 Move onto divisions with a remainder. 8 6 r 2 5 4 3 2 Finally move into decimal places to divide the total accurately. 1 4 6 16 21 3 5 5 1 1 . 0